

EFFECT OF SINGLE SCATTER PHASE FUNCTION DISTRIBUTION ON RADIATIVE TRANSFER IN ABSORBING-SCATTERING LIQUIDS

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Abstract—Calculations have been performed to determine the effect of the scattering distribution on radiation transfer in an absorbing-scattering liquid which is irradiated across an air interface. The scattering distribution has been systematically varied by changing the asymmetry factor used in the Henyey-Greenstein form of the phase function, and calculations have been based on the discrete ordinate, forward scattering and three-flux methods of solving the equation of transfer. A change in the scattering distribution from one which is highly peaked in the forward direction to one which is isotropic has the effect of increasing radiation absorption in the surface layers of the liquid, as well as the overall reflectance of the liquid. The three-flux predictions agree well with those of the discrete ordinate method over a wide range of scattering distributions, while there is poor agreement between the discrete ordinate and forward scattering results.

NOMENCLATURE

a_i ,	weights of Lobatto integration;
d ,	liquid layer depth [m];
F ,	net spectral radiation flux in liquid [W m ⁻² μm ⁻¹];
F^{*c} ,	spectral collimated flux incident on air-liquid interface [W m ⁻² μm ⁻¹];
F^{*d} ,	spectral diffuse flux incident on air-liquid interface [W m ⁻² μm ⁻¹];
F^{*-} ,	spectral flux leaving liquid in negative z direction at air interface [W m ⁻² μm ⁻¹];
g ,	asymmetry factor in Henyey-Greenstein phase function;
H ,	spectral volumetric absorption [W m ⁻³ μm];
I ,	spectral radiance [W m ⁻² sr ⁻¹ μm ⁻¹];
M ,	number of discrete ordinates;
N ,	number of terms used in phase function expansion;
p ,	spectral phase function;
P_k ,	Legendre polynomial of order k ;
R ,	overall spectral reflectance of liquid-bottom complex;
$S_{i,c}$,	collimated radiance scattering fraction;
$S_{i,j}$,	noncollimated radiance scattering fractions;
z ,	distance from air-liquid interface [m].

Ω ,	solid angle [sr ⁻¹];
ω ,	spectral single scatter albedo;
$\tilde{\omega}_k$,	weights of Legendre polynomials.

Subscripts

c ,	collimated;
d ,	bottom ($z = d$) condition.

Superscripts

$*$,	air side of air-liquid interface.
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INTRODUCTION

THE DIRECTIONAL distribution of the single scatter phase function can strongly influence the radiation field in an absorbing-scattering liquid which is irradiated in the visible portion of the spectrum. This influence is pertinent to the selection of methods for predicting the radiation field and to the selection of scattering agents for achieving desired radiation deposition or reflectance characteristics. It also determines the extent to which uncertainties in knowledge of the phase function, particularly for the backward direction, affect the accuracy of radiation field predictions.

To date, predictions of radiation transfer in absorbing-scattering liquids have largely considered isotropic scattering [1] or anisotropic scattering which is highly peaked in the forward direction [2-4]. Although these scattering distributions are limiting cases for the wide range of possible distributions, no effort has been made to consider the effect of this distribution on the radiation field. Accordingly, a major objective of this study has been to systematically vary the scattering distribution and to determine its effect on the radiation field for representative optical depths and scattering albedos. For each of the distri-

Greek symbols

β ,	spectral extinction coefficient [m ⁻¹];
θ ,	polar angle [rad];
θ_{crit} ,	polar angle for total internal reflection [rad];
μ ,	cos θ ;
ξ ,	relative scattering angle [rad];
ρ ,	spectral reflectivity;
τ ,	spectral optical depth (βz);

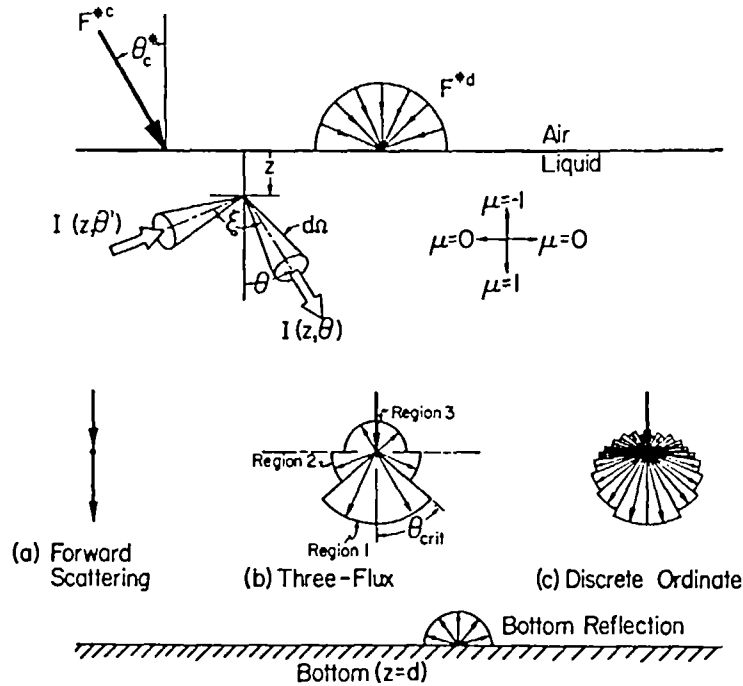


FIG. 1. Coordinate systems for radiation absorption and scattering in an aqueous medium.

bitions, predictions of the radiation field have been made by using the method of discrete ordinates. The ability of this rigorous method to provide accurate predictions for anisotropically scattering, aqueous media has been demonstrated [5, 6]. In addition, calculations have been performed to determine the effects of scattering distribution on the accuracy of the more approximate forward scattering [2] and three-flux [3] prediction methods, which have also been used for aqueous media.

RADIATION MODELS

Radiation transfer in an aqueous suspension may be represented by 1-dim. transfer through the planar, scattering-absorbing medium of Fig. 1. Radiation incident on the air-liquid interface may consist of collimated and diffuse components, which are reflected and refracted at the interface. Assuming homogeneous optical properties, an azimuthally symmetric radiation field, and negligible emission and polarization effects, the appropriate form of the quasi-steady equation of transfer is [7, 8]

$$\mu \frac{dI(\tau, \mu)}{d\tau} = I(\tau, \mu) + (\omega/2) \int_{-1}^{+1} p(\mu' \rightarrow \mu) I(\tau, \mu') d\mu' \quad (1)$$

where I , τ , ω and p are the spectral radiance, optical depth, single scatter albedo, and scattering phase function, respectively. The spectral net radiant flux in the liquid may be determined from knowledge of the radiance

$$F(\tau) = \int_{-1}^{+1} I(\tau, \mu) \mu d\mu \quad (2)$$

and may be used to determine the local volumetric absorption rate

$$H(\tau) = -\frac{dF}{dz} = -\beta \frac{dF(\tau)}{d\tau} \quad (3)$$

In normalized form, the net flux and volumetric absorption may be expressed as

$$\bar{F}(\tau) = F(\tau)/F^* \quad (4)$$

and

$$\bar{H}(\tau) = -\beta (d[\bar{F}(\tau)/F^*]/d\tau) \quad (5)$$

Because it determines the rate of thermal energy generation within the liquid, the volumetric absorption rate is of special interest in engineering applications. Another quantity of interest is the overall spectral reflectance of the liquid layer, which may be expressed as

$$R = F^*/F^* \quad (6)$$

The procedures used to solve equation (1) include the forward scattering model, the three-flux model, and the method of discrete ordinates. The forward scattering model is based on the approximation that radiation scattering is exclusively in the forward direction (Fig. 1(a)). That is, the scattered radiation is assumed to retain the original direction of propagation. Hence, regardless of the actual scattering distribution, the phase function is approximated by the following Dirac delta function

$$p(\mu' \rightarrow \mu) = 4\pi\delta(\mu' - \mu) \quad (7)$$

The method has been found to be suitable for scatter-

ing distributions which are highly peaked in the forward direction, and details of its development are provided elsewhere [2, 4].

The three-flux model represents a higher level of sophistication and is suggested by the fact that radiation transmitted into a liquid across an air interface is concentrated within the critical angle, θ_{crit} , associated with refraction at the interface. Since radiation may be scattered from this region into the remaining portion of the forward hemisphere ($\theta_{\text{crit}} < \theta \leq \pi/2$), as well as into the backward hemisphere ($\pi/2 < \theta \leq \pi$), it is reasonable to separate the radiation field into three isotropic components corresponding to the three regions shown in Fig. 1(b). Averaging equation (1) over each of the three regions, a system of three coupled, linear, ordinary differential equations may be obtained for the radiation flux components associated with each region [3, 4]. The equations are expressed in terms of the scattering parameters $S_{i,j}$ and $S_{i,c}$, which represent fractions of the noncollimated radiation in region j and the collimated radiation, respectively, which are scattered into region i . The parameters are defined as

$$S_{i,j} = \int_i \int_j \sum_{k=1}^N \tilde{\omega}_k P_k(\mu) P_k(\mu') d\mu d\mu', \quad (8)$$

$$S_{i,c} = \int_i \sum_{k=1}^N \tilde{\omega}_k P_k(\mu_c) P_k(\mu') d\mu' \quad (9)$$

where the integrations over regions i and j are confined to the range of μ associated with the region. Equations (8) and (9) are obtained by expanding the phase function in a Legendre polynomial series about the scattering angle and applying the theorem of spherical harmonics [3].

The discrete ordinate method divides the radiation field into any number M of discrete streams (Fig. 1 (c)), thereby allowing for a highly detailed solution to the equation of transfer. Expanding the phase function in a series of Legendre polynomials about the scattering angle and replacing the scattering integral in equation

(1) by a Lobatto quadrature formula, a system of M coupled, linear, nonhomogeneous differential equations of the following form may be obtained

$$\begin{aligned} \mu_i \frac{dI_i(\tau, \mu_i)}{d\tau} = & -I_i(\tau, \mu_i) + (\omega/2) \sum_{i=1}^M I_i(\tau, \mu_i) a_i \\ & \times \sum_{k=0}^N \tilde{\omega}_k P_k(\mu_i) P_k(\mu_k) \\ & + (\omega/2) [1 - \rho^* (\mu_c^*)] \\ & \times (\mu_c^*/\mu_c) F^{*c} \exp(-\tau/\mu_c) \\ & \times \sum_{k=0}^N \tilde{\omega}_k P_k(\mu_i) P_k(\mu_c), \quad 1 \leq i \leq M. \end{aligned} \quad (10)$$

Procedures for solving this system of equations are described by Houf [7].

To systematically examine the effect of changes in the scattering distribution, the phase function is based on the Henyey-Greenstein [9] result of the form

$$p(\cos \xi) = \frac{1 - g^2}{(1 + g^2 - 2g \cos \xi)^{3/2}} \quad (11)$$

where g is an asymmetry factor which may be varied from 0 (isotropic scattering) to 1 (strictly forward scattering). Phase function distributions corresponding to the values of g considered in this study are shown in Fig. 2. To use these distributions with the three-flux and discrete ordinate methods, however, it is necessary to represent the phase function as a series expansion of Legendre polynomials. A suitable expansion is of the form [10]

$$p(\xi) = \sum_{k=0}^N (2k+1) g^k P_k(\cos \xi) \quad (12)$$

where a value of $N = 150$ has been shown to accurately represent asymmetric functions [3] and was therefore used for this study.

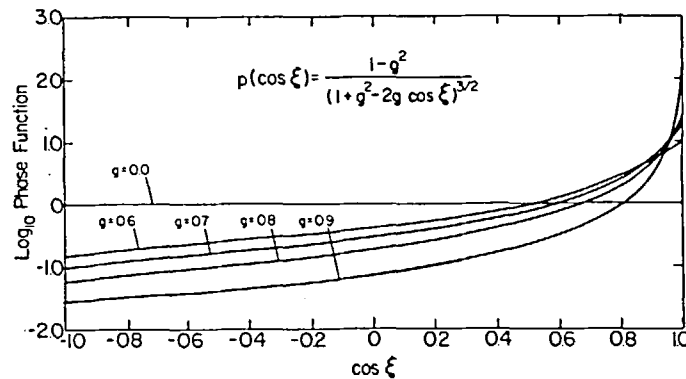


FIG. 2. Henyey-Greenstein phase function distributions.

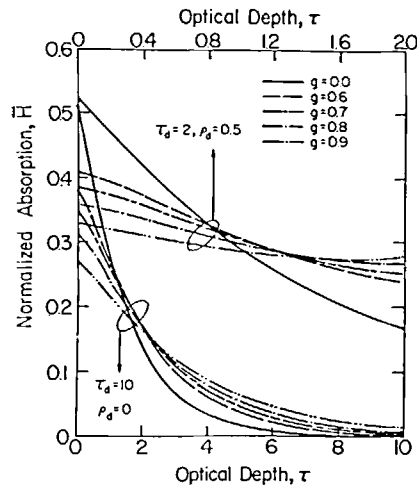


FIG. 3. Normalized volumetric absorption as a function of optical depth for diffuse irradiation and $\omega = 0.8$ (discrete ordinate solution).

RESULTS

To determine the effect of the asymmetry factor, and therefore the scattering distribution, calculations were performed for a range of liquid layer conditions. In one case the layer was treated as highly opaque ($\tau_d \equiv \beta d = 10$) with a nonreflecting bottom ($\rho_d = 0$), while in a second case the opacity was greatly reduced ($\tau_d = 2$) and diffuse bottom reflection was allowed to occur ($\rho_d = 0.5$). Both highly scattering ($\omega = 0.8$) and weakly scattering ($\omega = 0.2$) liquids were considered, and the asymmetry factor was varied from $g = 0$ (isotropic scattering) to $g = 0.9$ (highly forward peaked scattering). Diffuse and collimated irradiation conditions were considered, and reflection and refraction at the air-liquid interface were described by the Fresnel and Snell laws, respectively. The liquid was assumed to have an index of refraction of 1.33. Results are compared in terms of the normalized volumetric absorption and the overall reflectance defined by equations (5) and (6), respectively.

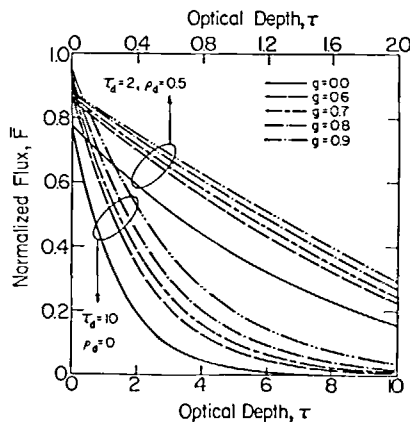


FIG. 4. Normalized flux as a function of optical depth for diffuse irradiation and $\omega = 0.8$ (discrete ordinate solution).

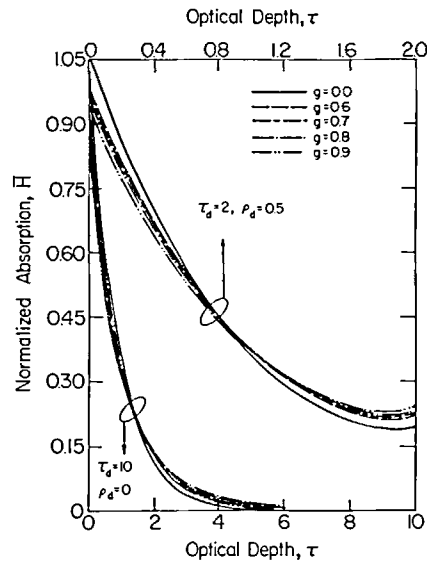


FIG. 5. Normalized volumetric absorption as a function of optical depth for diffuse irradiation and $\omega = 0.2$ (discrete ordinate solution).

Using results obtained from the discrete ordinate method of solution, the variation of the normalized volumetric absorption with optical depth is shown in Fig. 3 for diffuse irradiation and a single scatter albedo of $\omega = 0.8$. For both optical depths, the effect of the scattering distribution is significant. With increasing directional redistribution of the radiation due to scattering (decreasing g), there is greater radiation absorption in the upper layers of the liquid and less absorption in the lower layers. This trend is due to an increase in the radiation pathlength associated with scattering from forward to sideways and backward directions. If the pathlength for radiation transfer in a liquid layer increases, the amount of radiation absorbed in the layer will also increase. Hence, in the upper layers, for which the available radiant energy is

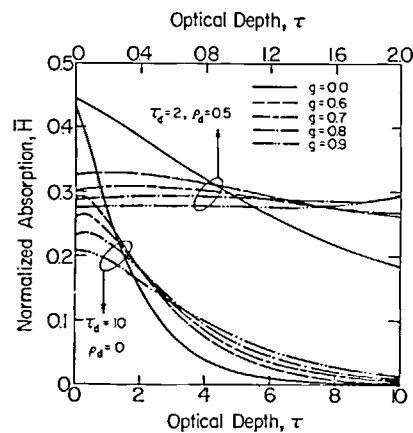


FIG. 6. Normalized volumetric absorption for collimated irradiation ($\theta_c^* = 30^\circ$) and $\omega = 0.8$ (discrete ordinate solution).

approximately independent of g , the volumetric absorption increases with decreasing g . In the lower layers, however, where the available radiant energy has been depleted by backscattering and absorption in the upper layers, the volumetric absorption decreases with decreasing g . These trends are consistent with those of the net flux, which are shown in Fig. 4. The gradient of this flux, $-d\bar{F}/d\tau$, increases with decreasing g for the smaller optical depths, but decreases with decreasing g for larger depths. The reduction in the net flux which occurs with decreasing g for all optical depths is due primarily to the increasing significance of backscatter.

Contrasting the results for $\tau_d = 2$ and $\tau_d = 10$ in Fig. 3, it is evident that, for $\tau \leq 2$, there is increased absorption for the $\tau_d = 2$ case. This result is due to the effect which bottom reflection has on increasing the amount of radiation available for absorption. Note from Fig. 4 that bottom reflection also has the effect of decreasing the net flux throughout the region $\tau \leq 2$, although the trend becomes less pronounced with decreasing g . The increase in absorption which occurs near the bottom for $\tau_d = 2$ becomes more pronounced with increasing g and may be attributed to the effect which bottom reflection has on increasing the amount of radiation available for absorption.

Results obtained for the volumetric absorption with $\omega = 0.2$ are shown in Fig. 5. Although the same trends revealed in Fig. 3 are in evidence, the effect of the scattering distribution is not pronounced. The results suggest that in a strongly absorbing medium, such as an ink suspension for which $\omega \lesssim 0.2$ [11], satisfactory predictions may be made by assuming isotropic scattering or strictly forward scattering, irrespective of the actual scattering distribution.

Calculations were also performed for the case of collimated irradiation, and representative results are shown in Fig. 6 for an incidence angle of $\theta_c^* = 30^\circ$ and a scattering albedo of $\omega = 0.8$. The effect of the scattering distribution is again pronounced and similar in form to that obtained for the diffuse irradiation. However, the manner in which absorption varies with optical depth differs for all but the case of isotropic scattering. In particular, for $g > 0$, the volumetric absorption is not a maximum at the interface but at some distance from the interface. Since the collimated flux undergoes a simple exponential decay [4], the existence of this maximum cannot be attributed to the absorption of collimated radiation. Instead, it is due to radiation which has been scattered from the collimated beam and is absorbed in a complicated manner which depends on the directional nature of the scattering. The increase in absorption which occurs at the bottom for $\tau_d = 2$ may again be attributed to bottom reflection effects. Note that, for $\tau_d = 2$, the existence of a highly forward peaked phase function causes the absorption to be nearly uniform throughout the liquid.

Results obtained for the overall reflectance, R , of the liquid-bottom complex are shown in Table 1 for the case of diffuse irradiation. The reflectance increases

Table 1. Overall reflectance values for a diffusely irradiated liquid (discrete ordinate solution)

g	$\tau_d = 10$	$\rho_d = 0$	$\tau_d = 2$	$\rho_d = 0.5$
	$\omega = 0.2$	$\omega = 0.8$	$\omega = 0.2$	$\omega = 0.8$
0	0.048	0.215	0.051	0.225
0.6	0.040	0.111	0.038	0.147
0.7	0.032	0.090	0.037	0.136
0.8	0.031	0.068	0.036	0.128
0.9	0.030	0.046	0.035	0.127

with decreasing optical depth and increasing bottom reflectivity. It also increases as the amount of radiation scattered in the backward direction increases (increasing ω and decreasing g). The effect which the scattering distribution has on the reflectance is most pronounced for highly opaque liquids of large albedo. Similar trends were obtained from calculations performed for collimated irradiation, with the magnitude of the reflectance depending strongly on the angle of incidence.

Although the existence of a backward peaked phase function is not a characteristic of liquid suspensions, it is of interest to consider the effect of such a hypothetical phase function on radiation deposition within the suspension. Accordingly, calculations were performed for $g = -0.9$, which corresponds to highly peaked backward scattering. In all cases the distribution of volumetric absorption with optical depth is similar to that obtained for isotropic scattering, except that, for large albedo ($\omega \gtrsim 0.8$), the magnitude of the absorption is everywhere smaller. This result is primarily due to a significant increase in the overall reflectance of the suspension, leaving less radiant energy for absorption. For small albedo ($\omega \lesssim 0.2$), results are within a few percent of those associated with isotropic scattering.

The accuracy of predictions based on the forward scattering and three-flux models may be assessed by comparing results with those obtained from the discrete ordinate method. Such a comparison is made in

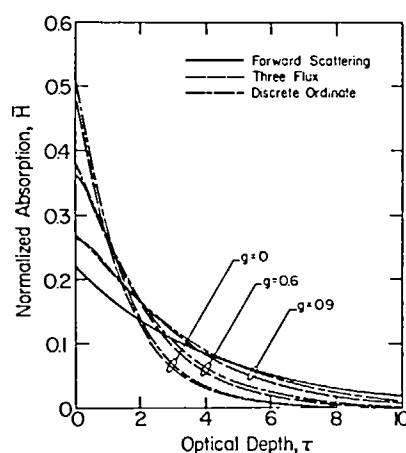


FIG. 7. Comparison of normalized volumetric absorption predictions for diffuse irradiation with $\tau_d = 2$, $\rho_d = 0.5$ and $\omega = 0.8$.

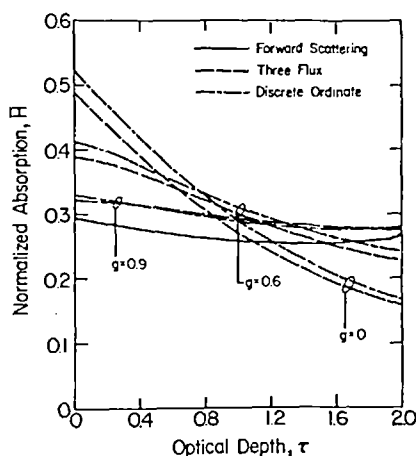


FIG. 8. Comparison of normalized volumetric absorption predictions for diffuse irradiation with $\tau_d = 2$, $\rho_d = 0.5$ and $\omega = 0.8$.

Fig. 7 for diffuse irradiation with $\tau_d = 10$, $\rho_d = 0$, $\omega = 0.8$, and values of $g = 0, 0.6$ and 0.9 . Since the forward scattering model is independent of the phase function distribution, a single result is obtained, irrespective of the value of g . It is therefore not surprising that this result agrees poorly with predictions based on the discrete ordinate method and that disparities increase with decreasing g . In contrast, agreement between the discrete ordinate and three-flux predictions is excellent.

The foregoing trends are also revealed in Fig. 8 for the shallow liquid layer, and additional calculations indicate that the trends are applicable to collimated, as well as diffuse, irradiation. With decreasing albedo, better agreement is achieved between the forward scattering and discrete ordinate results, with excellent agreement having been found to exist for $\omega \lesssim 0.2$.

SUMMARY

Calculations have been performed to determine the effect of the scattering distribution on radiation transfer in an absorbing-scattering medium which is irradiated across an air interface. The scattering distribution has been systematically varied by changing the asymmetry factor used in the Henyey-Greenstein form of the phase function. For moderate to large values of the albedo, backscattering becomes more pronounced and there is progressively more radiation absorbed in upper layers of the liquid, as the scattering distribution changes from one which is highly peaked in the forward direction to one which is isotropic. For small values of the albedo, $\omega \lesssim 0.2$, the effect of the scattering distribution on radiation absorption in the liquid may

be neglected. The overall reflectance of the liquid-bottom complex increases as the scattering distribution approaches isotropic conditions, with the effect being most pronounced for highly opaque liquids of large albedo.

Calculations have also been performed to determine the effect of scattering distribution on the accuracy of using the forward scattering and three-flux methods of prediction the radiation field. The accuracy of the forward scattering method decreases as isotropic scattering is approached and, for large albedo, is even marginal for scattering distributions which are highly peaked in the forward direction. In contrast, the accuracy of the three-flux method is excellent for the complete range of scattering distributions.

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EFFET D'UNE DISTRIBUTION DE FONCTION DE PHASE DE DIFFUSION SUR LE TRANSFERT RADIATIF DANS LIQUIDES ABSORBANTS ET DIFFUSANTS

Résumé—Des calculs sont conduits pour déterminer l'effet de la distribution de diffusion sur le transfert radiatif dans un liquide absorbant et diffusant qui est irradié à travers une interface d'air. La distribution de diffusion a été systématiquement variée en changeant le facteur d'asymétrie utilisé dans la forme de fonction de phase selon Henyey-Greenstein, et des calculs ont été basés sur l'ordonnée discrète dans le sens de la diffusion et des méthodes à trois flux de résolution de l'équation de transfert. Un changement dans la distribution de diffusion, depuis une qui est fortement pointue dans la direction avant jusqu'à une autre qui est isotrope, a l'effet d'accroître l'absorption du rayonnement dans les couches de surface du liquide, aussi bien que la réflectance globale du liquide. Les prédictions à trois flux s'accordent bien avec celles de la méthode de l'ordonnée discrète sur un large domaine de distributions de diffusion, tandis qu'un faible accord est remarqué entre les résultats par l'ordonnée discrète et la diffusion en avant.

DER EINFLUSS DER EINFACHEN PHASENSTREUUNGSFUNKTIONSVERTEILUNG AUF DIE STRAHLUNGSÜBERTRAGUNG IN ABSORBIERENDEN, STREUENDEN FLÜSSIGKEITEN

Zusammenfassung—Zur Bestimmung des Einflusses der Streuerverteilung auf den Wärmeaustausch durch Strahlung in absorbierenden und streuenden Flüssigkeiten, die durch eine Luftschicht angestrahlt werden, wurden Berechnungen durchgeführt. Die Streuerverteilung wurde systematisch verändert durch Variation des Asymmetriefaktors für die Henyey-Greenstein Form der Phasenfunktion, die Berechnungen basieren auf der diskreten Ordinate mit Vorwärts-Streuung und Dreistrommethoden für die Lösung der Wärmeübertragungsgleichungen. Eine Veränderung in der Streuerverteilung um eine Einheit, welche sehr schnell zur isotropen Flüssigkeit führt, hat den Effekt einer verstärkten Strahlungsabsorption an der Oberfläche der Flüssigkeit als auch auf die gesamte Reflexion der Flüssigkeit. Die Prognosen der Dreistrommethode stimmen sehr gut mit der diskreten Ordinatenmethode für einen breiten Streubereich überein, dagegen ist die Übereinstimmung zwischen der diskreten Ordinaten und der Vorwärts-Streuung nicht gut.

ВЛИЯНИЕ РАСПРЕДЕЛЕНИЯ ФУНКЦИИ ЕДИНИЧНОЙ РАССЕИВАЮЩЕЙ ФАЗЫ НА ЛУЧИСТЫЙ ПЕРЕНОС В ПОГЛОЩАЮЩИХ-РАССЕИВАЮЩИХ ЖИДКОСТЯХ

Аннотация—Выполнены расчеты для определения влияния формы распределения рассеянного излучения на лучистый перенос в поглощающей и рассеивающей жидкости, на которую падает излучение перпендикулярно границе раздела с воздухом. Распределение рассеянного излучения варьировалось за счет изменения коэффициента асимметрии, используемого в зависимости Хеней-Гринштейна для фазовой функции, а для решения уравнения переноса использовались методы дискретных ординат, прямого рассеивания и методы трех потоков. Изменение распределения рассеянного излучения от состояния, характеризующегося большим пиком в прямом направлении, до изотропного увеличивает поглощение излучения в поверхностных слоях жидкости, а также суммарную отражательную способность жидкости. Результаты расчетов методом трех потоков в широком диапазоне хорошо согласуются с расчетами методом дискретных ординат. В то же время отмечено плохое совпадение результатов по методу дискретных ординат и методу прямого рассеивания.